Teaching & Curriculum : Lesson & Unit Plans

Measuring the earth from a shadow

Topic : Geometry: Parallel lines cut by a transversal form congruent alternate interior angles.
Target Age: 8 – 15
Planning Framework: Romantic
Unit Length: 2-3 weeks
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Synopsis :

How can we measure the circumference of the earth? Even if we know it is round, how could we work out how far we would have to go to get back to where we are? We can get a clue about how to go about it from an ancient Greek, who managed to accurately measure the circumference of the earth by using some basic geometry and a stick. We can start with Eratosthenes's achievement and then replicate it using our own sticks and the internet.

Our local curriculum requires us to teach the theorem: "Parallel lines cut by a transversal form congruent alternate interior angles." How can the framework help in organizing this topic so that it will be imaginatively engaging to students? Let's lay it out category by category and see what we get.

1. Identifying "heroic" qualities

What heroic human qualities are central to the topic? What emotional images do they evoke? What within the topic can best evoke wonder?

In order to help students connect emotionally to the material, teachers need to first identify their own emotional attachment to it. What heroic human quality or emotion––courage, compassion, tenacity, fear, hope, loathing, delight, or whatever––can we identify in the topic of measuring the earth? These "romantic" qualities help us––and help our students––see the world in human terms and give human meaning to events, facts, and ideas in all disciplines. This first task is the most difficult part of planning the lesson or unit. We are asked to feel about the topic as well as to think about it. If our aim is to engage students' imaginations, we must first alert and exercise our own, and identify the transcendent qualities in the topic that provide a key to imaginative stimulation.

So how can we feel anything about congruent alternate interior angles? Not exactly a topic to set the blood pounding. But if we think of it for a while, we can locate some human source that can bring the topic to imaginative life. One heroic quality evident in this task comes from the practical ingenuity involved in using this theorem to measure the circumference of the earth, and this is best shown in the work of the person who first managed it with remarkable accuracy—the ancient Greek, Eratosthenes, head of the great library at Alexandria.

Identifying heroic qualities:

1. Main heroic quality: practical ingenuity

2. Alternative(s): the power of a simple theorem to achieve an astonishing result

Images that capture the heroic quality:
What in the world connected with congruent alternate interior angles can evoke the sense of practical ingenuity? I see, Eratosthenes, the venerable director of the library at Alexandria very carefully erecting a rod in a pit in a courtyard of the library, ensuring that it is exactly vertical, and then measuring the angle of the shadow cast by the sun. What he is doing, with this simple tool, is calculating the circumference of the earth.

2. Organizing the topic into a narrative structure

2.1 Initial access

What aspect of the topic best embodies the heroic qualities identified as central to the topic? Does this expose some extreme of experience or limit of reality?

For the first lesson of a unit or the opening part of a single lesson, teachers are asked to search their own imagination for images that catch the heroic quality that will provide the dramatic structure for the unit. Remember, it is as important to feel the heroic qualities as well as think about them. Rather than focus exclusively on the content and how we will organize that, we should also search our understanding of the topic and its content for those images that best capture what is important about it.

Initial access to the topic might usefully be provided in this case through the image we formed. In part anticipating the requirement that we "humanize" the content, we can begin by introducing the students to the formidable polymath Eratosthenes of Cyrene (c. 275 - 194 B.C.). He was one of those omnivorous inquirers with a distinct practical bent, in the style of Leonardo da Vinci. He did significant work in astronomy, history, literary criticism (including a 12-volume work, On Ancient Comedy), philosophy, poetry, and mathematics. He also devised a calendar, with leap years. He calculated the distances of the moon and the sun from the earth, though not as accurately as his calculation of the earth's circumference. In old age he became blind and, so the story goes, voluntarily starved himself to death. Such an introduction also can catch at some extremes of reality and experience. We will, then, structure the body of this short unit by beginning with Eratosthenes' method of calculating the circumference of the earth, seeing the theorem ingeniously applied to a practical problem

2.2 Composing the body of the lesson or unit

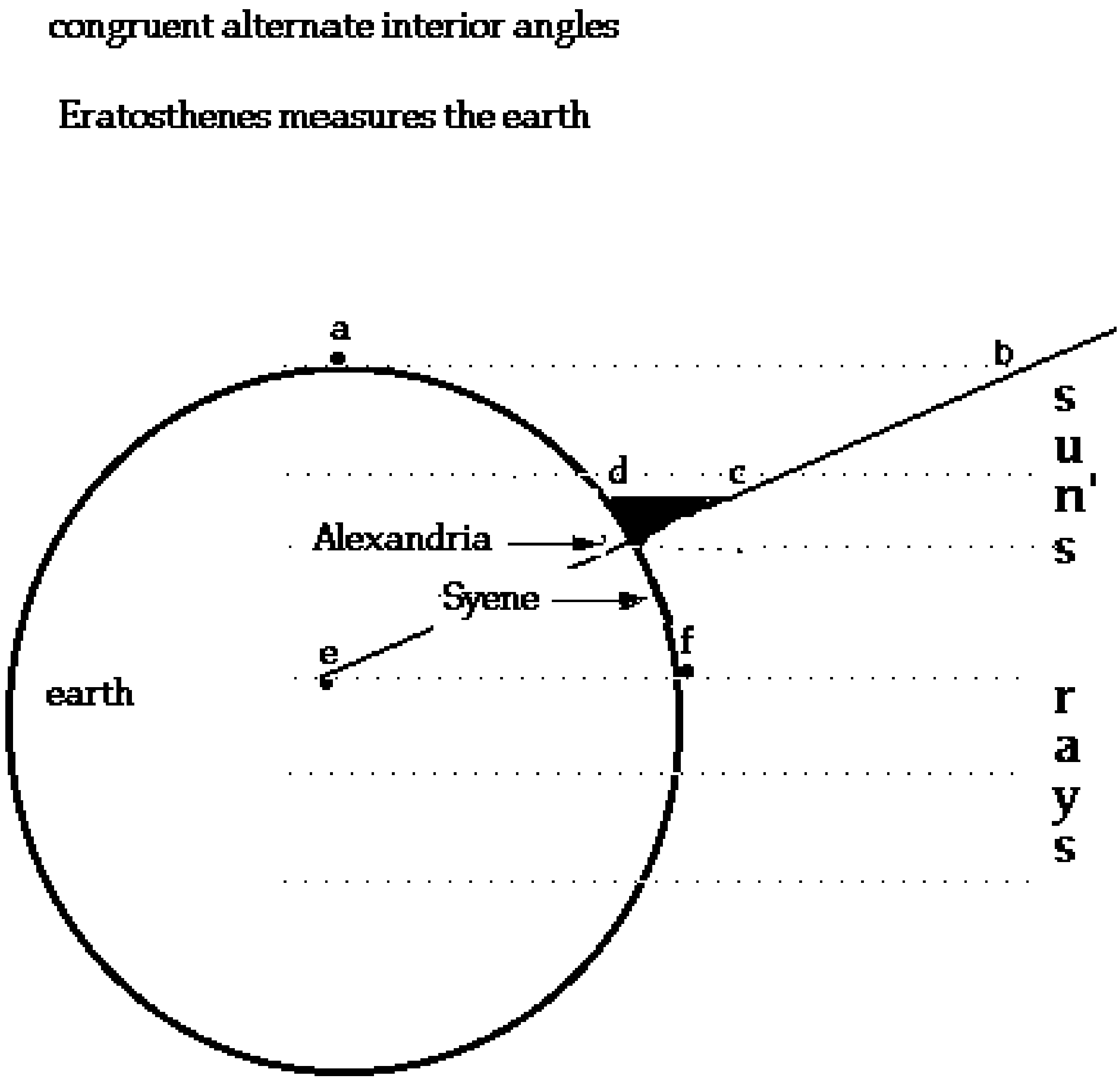
How do we organize the material into a story to best illustrate the heroic qualities? Sketch the story, ensuring that the qualities will be made clear by the narrative.

The principal heroic quality should provide the drama and conflict in the story. Remember, the heroic qualities should be those that most effectively convey the content of the topic. In making this brief initial sketch, try to capture just the main narrative thread that will carry the students' understanding from the beginning to the end of the lesson or unit.

Sketch the overall structure of the lesson/unit:
How did Eratosthenes use the theorem to calculate the earth's circumference using a vertical rod in Alexandria? Five hundred miles to the south of Alexandria, on what we call the Tropic of Cancer, was the town of Syene, on the site of present-day Aswan. (This calculation works out more neatly in miles than in kilometers.) Eratosthenes knew that at noon on the summer solstice in Syene a vertical rod cast no shadow. He also knew that the sun was very distant from the earth and that its rays could be considered as striking different places on the earth in parallel. So to our image. Around 200 B.C. Eratosthenes mounted a vertical rod in Alexandria and at noon on the summer solstice he measured the angle its shadow cast. From that measure he calculated with remarkable accuracy, by means of our theorem, the circumference of the earth. How?

Consider the diagram in Figure 1.

Figure 1



From the theorem Eratosthenes (with whose name students should obviously become familiar and they should be able to pronounce it accurately) knew that <ABE and <BEF in the above diagram are congruent alternate interior angles — i.e., whatever <ABE is, <BEF will be the same. By holding a pole upright in Alexandria he held it along the line EB. The top of the pole forms <ECD, which is also congruent with <ABE and <BEF. Eratosthenes' careful measurement of the angle of the shadow at noon — <ECD — yielded 7° 12'. That meant that <BEF must also be 7° 12'. 7° 12' is about 1/50 of the 360° of a full circle, and so the distance from Alexandria to Syene was 1/50 of the circumference of the earth. 50

x 500 = 25,000, so the circumference of the earth must be about 25,000 miles — which it is.

2.3 Humanizing the content

What aspects of the story best illustrate the human emotions in it and evoke a sense of wonder? What ideals and/or challenges to tradition or convention are evident in the content?

Think of how a good movie or novel makes aspects of the world engaging. Obstacles to the hero are humanized in one form or another, almost given motives; they are seen in human terms. To do this, we don't need to falsify anything, but rather we highlight a particular way of seeing it––because this is precisely the way students' imaginations are engaged by knowledge.

What content can be best shown in terms of hopes, fears, intentions or other emotions?

Humanizing the content should be relatively easy by extending the association with Eratosthenes' practical ingenuity. He exemplifies a magus-like cleverness. He did not need to travel around the world, measuring distance in the conventional ways. Using the ingenious techniques of geometry — a kind of magic too casually taken for granted — he was able to calculate the earth's circumference to within a few miles, at a time when most of its inhabitants had no idea even that it was round. We would do well to work at bringing to the fore the awe, wonder, and romance of geometry, rather than treating it as a set of routine, taken-for-granted techniques to be learnt. By associating with Eratosthenes's ingenuity in calculating distances to the moon and sun, and the earths' circumference, by extending from the slim base of the geometric theorem that was already known, one can help students become engaged by what is remarkable and wonderful about geometry. Geometry, and mathematics in general, can thus be seen, not just as an endless set of algorithms and theorems, but as a romantic adventure.

The convention that we can represent Eratosthenes revolting against is that which accepts the unknown as unknowable, which feels powerless in the face of whatever is beyond the boundary of established norms and routines. The ideal he represents is the determined use of practical ingenuity to achieve what to the conventional mind is impossible.

2.4 Pursuing details

What parts of the topic can students best explore in exhaustive detail?

While it is easy to give students a project to do that is part of a topic, it is a little harder to think about what aspect of the topic they might be able to exhaust, i.e. be able to find out nearly everything that is known about it. But there are such parts in every topic, and the security and sense of mastery that comes from knowing nearly as much as anyone about something is a great stimulus to inquiry. Think of something that is intriguing, that can be seen from a variety of different perspectives, or that is alluded to but not examined in detail in the content or in your teaching of it (referring to your notes from 2.2 and 2.3 above should help).

List those aspects of the topic that students can explore exhaustively:

In this case, we can have the students take part in a repetition of Eratosthenes's discovery. All they need is an internet connection with a school a known distance to the south of theirs and a stick they can erect, on a sunny day, exactly perpendicular, and then some means of measuring the angle of the shadow it casts. While it is easier to work out with cities on or within the tropics, one can still use this method with any two cities distant from each other that are on much the same longitude. Teachers in, say, Toronto and Miami, or Calgary and Phoenix, or Melbourne and Cooktown, could arrange to measure the angle cast by the sun at noon on particular days, compare figures, and, with slight modification, use Eratosthenes' method to calculate the earth's circumference. This might make a "re-embedding" activity that could conclude the narrative of the unit.
One could also let small groups of students explore all that is known about the great library at Alexandria, or about Eratosthenes himself. Students can discover all we know about these exotic topics in quite a short time of intensive exploration. They could also explore the lives and achievements of other mathematicians at the time. Or compare Eratosthenes's measures of the earth with the much less accurate ones reported by Ptolemy. (Ptolemy's became the more widely believed—which is why Columbus thought he had reached Asia when he reached land in the Caribbean.)

3. Concluding

How can one best bring the topic to satisfactory closure? How can the student feel this satisfaction?

One wants to end a topic in a "romantic" way, which can have two forms. The first form is to re-examine the images we started from and review the content through the lenses of other heroic qualities, including some that might give an opposite or conflicting image to that of our earlier choice. The second form is to show how the romantic association the student has formed can help them understand other topics in a new, more imaginative, way. Or one can use both, of course.

Concluding activities: Useful details that might elaborate this topic and the association with practical ingenuity might include a study of what is known of Eratosthenes' life and his other achievements; the lives and practical ingenuity of other Greek geometers; what is known of the library at Alexandria; other practical uses for the theorem; and so on.

4. Evaluating

How can one know that the content has been learned and understood and has engaged and stimulated students' imaginations?

Any of the traditional forms of evaluation can be used, but in addition, teachers might want to get some measure of how far students' imaginations have been engaged by the topic, how far they have successfully made a romantic engagement with the material. In addition, the concluding activities (above) are also evaluative in nature. Various kinds of information, including that derived from discussion, debate, artwork, journal writing, etc., can be gained as the unit is being taught. The teacher can also measure the amount of non-required reading students engage in. They might also record what other reading or video-watching, web-surfing they may have performed related to the subject matter of the topic. In addition they could ask the students to keep personal notes in which they record in an open-ended way any ideas they have had about the topic they are studying.

Forms of evaluation to be used:

One can use traditional forms of evaluation to discover whether students understand the geometrical principle and can apply it to new cases. Given our intention, we will also want to look for evidence that students have associated in some degree with the quality of practical ingenuity that Eratosthenes has exemplified for us. A difficulty with looking for precise measures of such qualities is that they may take different forms in each individual's behaviour. But the observant teacher would be able to use practical ingenuity to measure whether or not, or the degree to which, students find the discovery of congruence in alternate interior angles imaginatively engaging. At the simplest level, their enthusiasm would provide an index, as would their interest in learning about some mathematicians and geometers contemporary with Eratosthenes, such as Archimedes, Nicomedes, and others. I suspect that the results of the traditional evaluation procedures might also give some insight into students' association with practical ingenuity, as I would anticipate much better results from traditional tests after such a method of teaching, due simply to students' engagement by the topic.

Because we have been trying to engage students' imaginations with geometry, we will also want to evaluate how successful we have been in this regard. We do not have well-tried and tested evaluation procedures that will give us precise readings of imaginative engagement, and probably never will have. But we might experiment with plausible ways of getting some kind of reading. We might begin, simply, with teachers' observation. It is usually fairly clear whether or not students are imaginatively engaged in a topic; the degree of their enthusiasm, the way it invades their intellectual activity in general, their pursuit of aspects of it well beyond what is required, their questioning and searching out additional sources of information, their desire simply to talk about it, are all indicators of some degree of imaginative engagement. Students' written work, or other forms in which they present what they have learned to the teacher or to the class as a whole, can yield evidence of imaginative engagement; going beyond what is required, especially when the direction has been determined by the student's perhaps idiosyncratic interests, or taking great care in, for example, gathering data from their own and other schools' recording of their experiment, or evidence of knowledge that has been culled from diverse sources not readily available, or evidence of a kind of obsessive interest in some feature of geometers' lives, would all provide some indication of imaginative engagement. Some of the above characteristics of students' work could, of course, be due to other factors, like desire for a high grade or compulsion. But it is an unusually unobservant teacher who cannot tell the difference. These points echo in brief ideas that are elaborated and developed in Eisner's "connoisseurship" model of evaluation (Eisner, 1985).

This is an area in which one might encourage students' self-evaluation. Ask them to reflect on how far they felt they had been imaginatively engaged in the topic, what features of it engaged them most, what had they most enjoyed learning about, and so on. This might also become a useful small group activity, in which each other's interests might incidentally be communicated to the group.

A part of the attempt to evaluate a unit such as this must involve trying to discover how far students grasp the underlying scientific virtue of pursuing knowledge purely for its own sake, and recognizing practical ingenuity as appropriately serving this pursuit. Also we will want to evaluate how far students associate with this human quality. We can try to get some reading of these from students' work, from their classroom behavior, and from effects on what they more readily turn their minds to in leisure time. The sensitive teacher will no doubt be able to get an adequate reading on their success, even though it will not be in terms of some precise score. (We decide what it is educationally valuable to do on grounds other than what we can evaluate precisely.)

There has been a considerable development in recent years of what are generally called "qualitative evaluation" procedures. Many of these would be useful here. A clear introduction and discussion of some of these procedures is available in Schubert (1986), and more elaboration and detail is available in Guba and Lincoln (1981) and in Patton (1990).

Concluding note: Lest you feel awed by my massive erudition, let me confess that it took me perhaps as much as an hour to find an historical example to illustrate the theorem. Then I spent about twenty minutes looking up Eratosthenes in an encyclopedia and a classical dictionary. This easily gathered knowledge provided the framing for the stark theorem that can serve to humanize it and embed it in an accessibly meaningful context. (I originally used this example in my book <u>Imagination in Teaching and Learning</u>, published in 1992. I have discovered since then that Eratosthenes has become almost a standard “topic” in teaching this theorem. Heigho.)